# C.U.SHAH UNIVERSITY <br> Winter Examination-2015 

## Subject Name: Design and Analysis of Algorithms Subject Code: 4TE05DAA1

## Semester: 5 Date:7/12/2015 Time: 2:30 To 5:30 Marks: 70

Instructions:
(1) Use of Programmable calculator \& any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

## Q-1 Attempt the following questions:

a) Define Algorithm.
b) Which of the following is not $\mathrm{O}\left(\mathrm{n}^{2}\right)$ ?
i). $n+10000 n$
ii). $\mathrm{n}^{1.9999}$
iii). $10^{5} n+2^{6} n$
iv). $\mathrm{n}^{3} / \sqrt{ } \mathrm{n}$
c) The number of comparisons done by sequential search is $\qquad$
i) $(\mathrm{N} / 2)+1$
ii) $(\mathrm{N}+1) / 2$
iii) $(\mathrm{N}-1) / 2$
iv) $(\mathrm{N}+2) / 2$
d) Two main measures for the efficiency of an algorithm are
i). Processor and memory
ii). Complexity and capacity
iii). Time and space
iv). Data and space
e) The quick sort algorithm exploit
i). Greedy
iii). Divide and Conquer
$\qquad$ design technique
ii). Dynamic programming
iv). Backtracking
f) A list of $n$ strings, each of length $n$, is sorted into lexicographic order using the mergesort algorithm. The worst case running time of this computation is
i). $O(n \log n)$
ii). $O\left(n^{2} \log n\right)$
iii). $O\left(n^{2}+\log n\right)$
iv). $\mathrm{O}\left(\mathrm{n}^{2}\right)$
g) Let $\mathrm{f}(\mathrm{n})$ and $\mathrm{g}(\mathrm{n})$ be two asymptotically positive functions. Prove or disprove the $2^{2 \mathrm{n}}=$ $\mathrm{O}\left(2^{\mathrm{n}}\right)$ (using the basic definition of $\mathrm{O}, \Omega$ and $\Theta$ ).
h) ${ }_{7} \mathrm{C}_{5}=$ ? (Hint: Binomial Co-efficient)
i) The Knapsack problem belongs to the domain of $\qquad$ problems.
i) Optimization
ii) NP Complete
iii) Linear Solution
iv) Sorting
j) Consider the following function f : int $f($ int $n)$
\{
int s $=0$;
while ( $\mathrm{n}>1$ )
\{


```
n = n/2;
s++;
}
return s;
}
```

What is the asymptotic complexity in terms of n ? (Pick the smallest correct answer)
i). O ( $\mathrm{n} \log \mathrm{n}$ )
ii). O (n)
iii). $O(\log n)$
iv). $O\left(\mathrm{n}^{2}\right)$
k) Define Asymptotic notation.
l) Can Master Theorem be applied to the recurrence of $T(n)=4 T(n / 2)+n^{2} \lg n$ ? Why and why not? Give an asymptotic upper bound of the recurrence?
m) List the characteristics of algorithm.
n) Which of the following sorting methods would be most suitable for sorting a list which is almost sorted?
i). Merge Sort
ii). Insertion Sort
iii). Selection Sort
iv). Quick Sort

## Attempt any four questions from $\mathbf{Q}-2$ to $\mathbf{Q - 8}$

Q-2 Attempt all questions
(a) Using recurrence tree method solve the following recurrences:
(i) $\mathrm{T}(\mathrm{n})=\mathrm{T}(\mathrm{n} / 3)+\mathrm{T}(2 \mathrm{n} / 3)+\mathrm{O}(\mathrm{n})$
(ii) $\mathrm{T}(\mathrm{n})=3 \mathrm{~T}(\mathrm{n} / 4)+\mathrm{cn}^{2}$
(b) Solve the following recurrences:
(i) $T(n)=3 T(n / 4)+n \lg n$
(ii) $T(n)=T(n-1)+n$

Q-3 Attempt all questions
(a) Explain the single source shortest path algorithm. (Hint: Dijkastra's Algorithm)
(b) What do you mean by amortized analysis? Explain the techniques used in amortized analysis.
Q-4 Attempt all questions
(a) Explain the merge sort algorithm with an example.
(b) Generate the minimum spanning tree for the given graph using Kruskal's algorithm.

Also explain the algorithm.

(c) Given 10 activities along with their start and finish time as
$\mathrm{S}=\left\langle\mathrm{A}_{1}, \mathrm{~A}_{2}, \mathrm{~A}_{3}, \mathrm{~A}_{4}, \mathrm{~A}_{5}, \mathrm{~A}_{6}, \mathrm{~A}_{7}, \mathrm{~A}_{8}, \mathrm{~A}_{9}, \mathrm{~A}_{10}\right\rangle$

$\left.\mathrm{S}_{\mathrm{i}}=<1,2,3,4,5,6,7,8,9,10\right\rangle$
$\mathrm{F}_{\mathrm{i}}=\langle 5,3,4,6,7,8,11,10,12,13\rangle$
Compute a schedule where the largest numbers of activities take place.
Q-5

## Attempt all questions

(a) Find an optimal solution for the knapsack Instances $\mathrm{n}=7, \mathrm{M}=15$,
$(P 1, P 2, \ldots, P 7)=(10,5,15,7,6,18,3)$ and $(W 1, W 2, \ldots, W 7)=(2,3,5,7,1,4,1)$
(b) Write Prim's algorithm for minimum spanning tree. Generate the minimum spanning tree for the given graph.

(c) Write down an algorithm for insertion sort.

## Q-6 Attempt all questions

(a) Consider five items along their respective weights and values
$\mathrm{W}=\langle 5,10,20,30,40\rangle$ and $\mathrm{V}=<30,20,100,90,160\rangle$. The capacity of Knapsack $\mathrm{M}=60$.
Find the solution to the fractional Knapsack proble.
(b) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 4,10,3,12,20,7\rangle$.

## Q-7 Attempt all questions

(a) Explain KMP string matching algorithm. For the text $T=x y x x y x y x y y x y x y x y y x y x y x x y$ and pattern $\mathrm{P}=$ xyxyyxyxyxx find the value of function $\pi$.
(b) What is the difference between Greedy algorithms and Dynamic Programming?

Find out the longest common subsequences from the two given sequence of characters:
S1 = <A, B, C, D, B, C, D, C, D, D $>$
$\mathrm{S} 2=\langle\mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{C}, \mathrm{D}\rangle$

## Q-8 Attempt all questions

(a) What do you mean by P, NP, NP-complete and NP-Hard problems?
(b) Explain the Rabin-Karp string matching algorithm with an example.
(c) What is backtracking? Explain N -queen problem. Also give the solution for the 8 -Queen problem.


